Homework 7, due 4/10

1. Let X be a Kähler manifold with Kähler form ω. Show that ω is harmonic, i.e. $\bar{\partial}\bar{\partial}^*\omega + \bar{\partial}^*\bar{\partial}\omega = 0$.

2. Show that on a compact Kähler manifold, a $(p, 0)$-form is holomorphic if and only if it is harmonic.

3. Use the exact sequence of sheaves

$$0 \to \mathbf{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0,$$

and the facts that $H^1(\mathbf{P}^n, \mathbf{R}) = 0, H^2(\mathbf{P}^n, \mathbf{Z}) \cong \mathbf{Z}$ to show that $H^1(\mathbf{P}^n, \mathcal{O}^*) \cong \mathbf{Z}$.

4. Show that on a compact Kähler manifold X, the dimensions $\dim H^k(X, \mathbf{R})$ are even for all odd k.